Distance magic and group distance magic graphs

Sylwia Cichacz, Dalibor Froncek

July 20, 2016

A *distance magic labeling* (also called *sigma labeling*) of a graph $G = (V, E)$ of order n is a bijection $\ell\colon V \to \{1, 2, \dots, n\}$ with the property that there is a positive integer k (called the *magic constant*) such that

$$w(x) = \sum_{y \in N_G(x)} l(y) = k \text{ for every } x \in V(G),$$

where $w(x)$ is the *weight* of vertex x. If a graph G admits a distance magic labeling, then we say that G is a *distance magic graph*, see [1].

The notion of group distance magic labeling of graphs was introduced in [2]. Let G be a graph with n vertices and Γ an Abelian group with n elements. We call a bijection $g\colon V(G) \to \Gamma$ a Γ-*distance magic labeling* if for all $x \in V(G)$ we have $w(x) = \mu$ for some μ in Γ. A graph G is called a *group distance magic graph* if there exists a Γ-distance magic labeling for every Abelian group Γ of order $|V(G)|$.

Let G be a distance magic graph of order n. If we replace n in $\{1, 2, \dots, n\}$ by 0, we obtain a $\mathbb{Z}_n$-distance magic labeling. Thus, every graph with n vertices and a distance magic labeling also admits a $\mathbb{Z}_n$-distance magic labeling. The converse is not necessarily true (see, e.g., [2]). However, so far there is not known a distance magic graph that is not group distance magic.

Open problem: if G is a distance magic graph, then is G group distance magic?

References

[1] S. Arumugam, D. Froncek, N. Kamatchi, *Distance Magic Graphs—A Survey*, Journal of the Indonesian Mathematical Society, Special Edition (2011) 11–26.

[2] D. Froncek, *Group distance magic labeling of of Cartesian products of cycles*, Australasian Journal of Combinatorics 55 (2013) 167–174.