Constructions of universalized Sierpiński graphs based on labeling manipulations

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Sierpiński graphs are known to be graphs with self-similar structures, and their various properties have been investigated until now. Besides, they are known to be isomorphic to WK-recursive networks which have been proposed as interconnection networks because of their nice extendability. As a generalization (resp., variant) of Sierpiński graphs, generalized Sierpiński graphs (resp., extended Sierpiński graphs) have been introduced. In this talk, we newly define a larger graph class which includes both generalized Sierpiński graphs and extended Sierpiński graphs, and moreover uneven graphs with self-similar structures such as Fobonacci trees.

Let G be a simple undirected graph which may have a self-loop. The *universalized Sierpiński graph* $\Upsilon(G, n)$ is defined to be the graph with the vertex set consisting of all n-tuple $(v_1, v_2, \ldots, v_n)$ where $v_i \in V(G)$ for $1 \leq i \leq n$ and $v_i \neq v_{i+1}$ if v_i has no self-loop, and in which two vertices $(u_1, u_2, \ldots, u_n)$ and $(v_1, v_2, \ldots, v_n)$ are adjacent if and only if there exists an integer j such that $u_i = v_i$ for $1 \leq i < j$, $u_j v_j \in E(G)$, and $u_{j+\ell} = v_j, v_{j+\ell} = u_j$ for $1 \leq \ell \leq n - j$ if both u and v have a self-loop, $u_{j+\ell} = v_j, v_{j+\ell} = u_j$ (resp., $u_{j+\ell} = u_j, v_{j+\ell} = v_j$) for odd (resp., even) $1 \leq \ell \leq n - j$ otherwise. In particular, when G is a compete graph with a self-loop at each vertex, $\Upsilon(G, n)$ is a Sierpiński graph. Besides, if every vertex of G has a self-loop (resp., G is a complete graph), then $\Upsilon(G, n)$ corresponds to a generalized Sierpiński graph (resp., extended Sierpiński graph).

To construct $\Upsilon(G, n)$ based on the definition, we need to combine $|V(G)|$ copies of subgraphs of $\Upsilon(G, n-1)$. We then present an inductive algorithm whose essential parts consist of edge-labelings and vertex-labelings, to construct $\Upsilon(G, n)$ from the original graph G. An advantage of such an algorithmic construction of $\Upsilon(G, n)$ is to be able to investigate the structure of $\Upsilon(G, n)$ directly from that of $\Upsilon(G, n-1)$. We also present structural properties of universalized Sierpński graphs such as connectivity, vertex-colorings, edge-colorings, total colorings, hamiltonicity, and factorizations.

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