Handicap Tournaments

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A *round robin tournament of n teams* is a tournament in which every team plays the remaining $n-1$ teams. When the teams are ranked according to their strengths so that the strongest team is ranked 1 and the weakest is ranked n, we observe that the sum of rankings of all opponents of the i-th ranked team, denoted $w(i)$, is $w(i) = n(n+1)/2 - i$, and the sequence $w(1), w(2), \ldots, w(n)$ is a decreasing arithmetic progression with difference one. A tournament of n teams in which every team plays $kn-1$ opponents and the sequence $w(1), w(2), \ldots, w(n)$ is a decreasing arithmetic progression with difference one is called a *fair incomplete tournament*.

In such tournaments the best team plays the weakest opponents, while the weakest team plays the strongest opponents, which still favors the strongest teams. This is eliminated in *equalized incomplete tournaments* where the sum of rankings of all opponents of every team is the same.

Even then the weaker teams do not have the same chance of winning, simply because if a weak team plays the same opponents as a strong one, its total winning record will likely be worse. This can be avoided if the weakest teams play weak opponents, while the strongest teams play strong ones. That is, the sequence $w(1), w(2), \ldots, w(n)$ should be an increasing arithmetic progression with a common difference d. A tournament in which this condition is satisfied is called a *d-handicap incomplete tournament*.

Hence, a *handicap distance d-antimagic labeling* or shortly *d-handicap labeling* of a graph $G = (V, E)$ with n vertices is a bijection $f : V \to \{1, 2, \ldots, n\}$ such that $f(x_i) = i$ and the sequence $w(x_1), w(x_2), \ldots, w(x_n)$ forms an increasing arithmetic progression with difference d. When $d = 1$, the labeling is called just a *handicap labeling*. A graph G allowing such labeling is a *handicap distance d-antimagic graph* (or just *d-handicap graph*).

We will present some new classes of handicap distance 1-antimagic graphs with odd number of vertices. Our construction is based on a generalization of magic rectangles, called *regular semi-magic rectangle sets*.

We will also present a class of 2-handicap graphs of order $n \equiv 0 \pmod{16}$.

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